# Determination of a Mean Ballistic Coefficient and Decay Predictions for CubeSats

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## 1. Introduction

Satellites in Low Earth Orbit (LEO) decay due to some environmental effects on the orbit. The biggest effect for LEO orbits is atmospheric drag[1]. A simple equation to calculate decay of a satellite using atmospheric effects is given below.

$$\Delta a_{rev} = -2\pi \left(\frac{C_d A}{m}\right) \rho a \qquad (1)$$

Here, $\Delta a_{rev}$ is semi-major axis (SMA) change at one revolution around orbit, $C_d$ is drag coefficient of the satellite, $A$ is drag area, $m$ is mass of the satellite, $\rho$ is atmospheric density at the altitude and $a$ is semi-major axis. In general applications, selection of $C_d$ coefficient, drag area and a constant sun activity independent $\rho$, yields some errors. As a common approximation, $\left(\frac{m}{C_d A}\right)$ can be taken as ballistic coefficient $B$ of a satellite. . Some example satellites in the literature and their mean ballistic coefficients are given in the table below to show the readers change in ballistic coefficient from satellite to satellite[2].

Table 1. Ballistic Coefficients of Some Satellites

| Satellite | Mean Ballistic Coefficient |
|---|---|
| Oscar-1 | 29 |
| Echo-1 | 0.515 |
| ERS-1 | 73 |
| Skylab | 228 |

This paper investigates mean value of $B$ for 1U CubeSats using real flight data. Because CubeSats have similar dimension and mass properties, a mean generic ballistic coefficient prediction is more applicable for CubeSats. To improve the accuracy of calculated $B$, a method for determination of correct atmospheric density is developed and $B$ was calculated using the equation (1) and backward Two Line Element (TLE) data of available CubeSats.

## 2. Atmospheric Density Model

Solar activity is an important factor on atmospheric density. The first step was the determination of the solar activity at the desired date. In order to determine the solar activity, first, solar flux data and solar cycles between 1954 and 1996 were investigated.

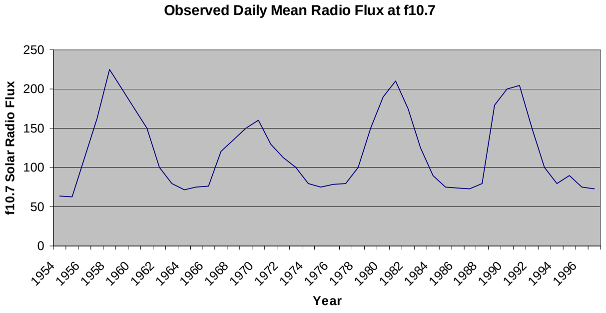

Figure 1. Observed Solar Flux Data Since 1954

These cycles were normalized to 100 to find the activity level percentage in a cycle. A polynomial is fitted to the data and the polynomial which is given in figure below was found. This polynomial gives the percentage of solar activity for a given date in its own cycle. This approach led to obtain a partially time independent model which can be used for a future predictions.

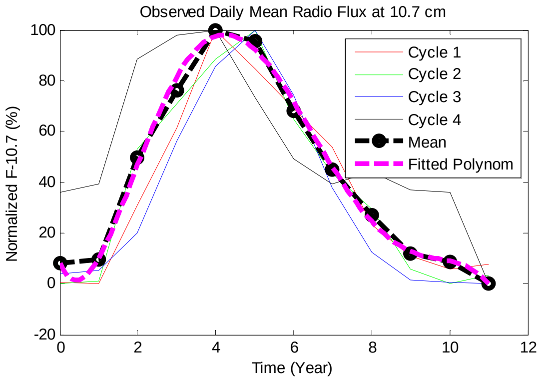

Figure 2. Processed Real Measured Solar Cycle Data

The polynomial coefficients are given in Figure 3 below.

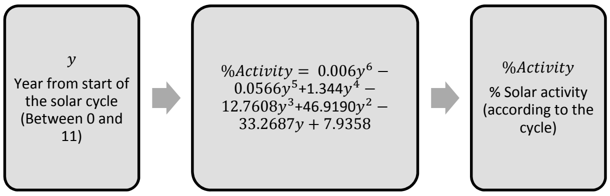

Figure 3. Fitted Solar Activity Polynomial

It was thought that %0 activity corresponds the minimum atmospheric density and %100 activity creates the maximum atmospheric density. Harris – Priester model[2] was taken as an atmospheric model because it is valid for altitude range of LEO and gives both minimum and maximum density values for a specific altitude. The real density depending on the solar activity ($\rho_{sa}$) level is:

$$\rho_{sa} = \frac{\rho_{max}-\rho_{min}}{100} \times (\%Activity) \tag{2}$$

Where $\rho_{max}$ is the maximum and $\rho_{min}$ is the minimum density which is computed via the Harris-Priester model for a certain altitude. The atmospheric density variation in Harris-Priester model is given on figure 4 below.

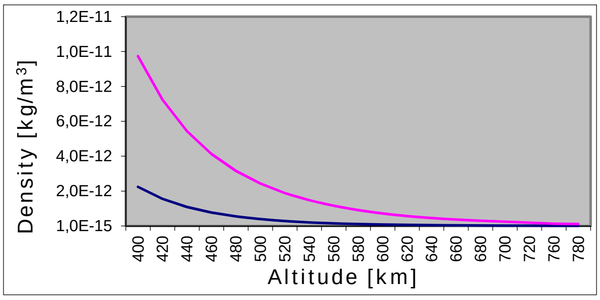

Figure 4 Harris-Priester Model Plot Showing Maximum and Minimum Atmospheric Densities

## 3. Determination of B

Backward TLE data was downloaded for 17 1U CubeSats and $B$ was solved as an optimization problem for each CubeSat. The problem is illustrated below.

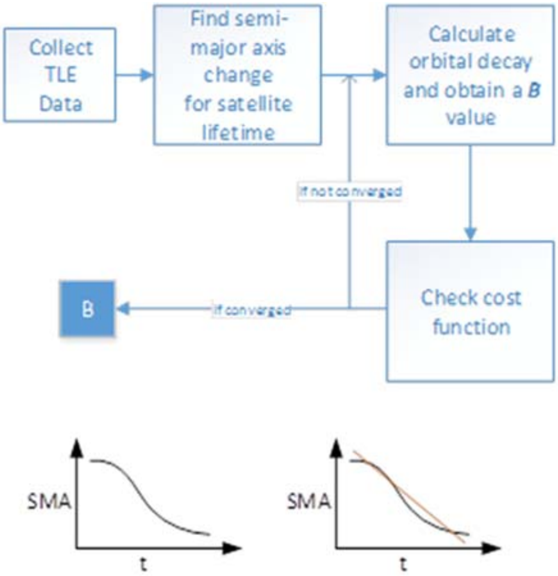

Figure 5. Analyses Steps for Ballistic Coefficient

Optimization cost function is created as below for the solution of B:

$$J = \frac{{e_0}^2+{e_1}^2+\cdots+{e_n}^2}{T} \tag{3}$$

Where $e_n$ is the error between real SMA data (Figure 6) and calculated SMA data (given in Figure 6) and $T$ is time. Dividing the cost functions by $T$ makes cost functions and fitting performances comparable between satellites that have different data and time span.

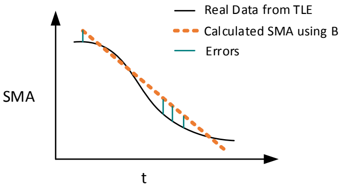

Figure 6. Semi-major Axis and Time Plot for Fitted and Real Data

As an example, decay analyses was performed for ITUPSAT-1 with new B which is obtained via the method proposed in this paper. The calculated SMA change and the real SMA change are given in Figure 7 below. In this calculation, $B = 6.10$ is the best value of $B$ for ITUPSAT-1. $\Delta a_{rev}$ was solved with a step size of 15 days by recalculating the new atmospheric density with respect to solar activity.

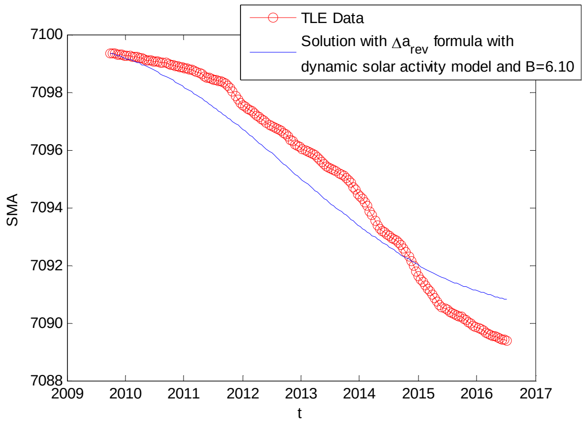

Figure 7. ITUPSAT-1 Semi-Major Axis Plot

## 4. Investigation of Results

The analysis in the above was done for all 1U CubeSats in this work. Satellites which completed their reentry were excluded, because atmospheric characteristics and dynamics are fluxional in altitudes near the Earth. The analysis results for CubeSats are summarized in Table 2 which contains calculated ballistic coefficient, satellite properties and orbital status.

Table 2. Analysis Results of 1U CubeSats

| Satellite Name | Size | Analysis Duration (Year) | Ballistic Coeff. | Initial Altitude | Launch Date | Decay Status | Normalized Cost |
|---|---|---|---|---|---|---|---|
| UWE3 | 1U | 2,61 | 92,59259 | 648,90 | 2013 | NO | 0,76 |
| ZACUBE-1 | 1U | 2,65 | 96,15385 | 641,09 | 2013 | NO | 0,83 |
| HINCUBE | 1U | 2,61 | 83,33333 | 640,63 | 2013 | NO | 0,94 |
| FUNcube-1 | 1U | 2,57 | 81,30081 | 640,60 | 2013 | NO | 0,95 |
| ESTcube-1 | 1U | 3,19 | 119,0476 | 667,21 | 2013 | NO | 1,33 |
| iCUBE-1 | 1U | 2,59 | 86,2069 | 617,12 | 2013 | NO | 1,43 |
| PUCPSat1 | 1U | 2,61 | 81,96721 | 616,95 | 2013 | NO | 1,49 |
| UWE-2 | 1U | 6,80 | 181,8182 | 719,39 | 2009 | NO | 1,54 |
| Libertad1 | 1U | 9,27 | 140,8451 | 722,33 | 2007 | NO | 1,66 |
| ITUpSAT1 | 1U | 6,80 | 163,9344 | 721,35 | 2009 | NO | 1,73 |
| HumSat-D | 1U | 2,62 | 66,22517 | 616,96 | 2013 | NO | 1,90 |
| Duchifat-1 | 1U | 2,09 | 69,44444 | 615,27 | 2014 | NO | 1,95 |
| SwissCube-1 | 1U | 7,30 | 147,0588 | 721,34 | 2009 | NO | 1,99 |
| CSTB1 | 1U | 9,28 | 117,6471 | 712,00 | 2007 | NO | 2,41 |
| BEESAT-2 | 1U | 3,22 | 78,125 | 569,86 | 2013 | NO | 8,17 |
| BEESAT-3 | 1U | 3,21 | 75,18797 | 637,09 | 2011 | NO | 8,61 |
| SOMP | 1U | 3,26 | 67,11409 | 569,96 | 2013 | NO | 9,90 |

The calculated mean value of ballistic coefficients is 102 and the standard deviation is 35. At the next step of this study, an analysis is performed to investigate the profit of using the mean value of ballistic coefficient. To do this, all analyses for CubeSat satellites were performed again with the mean value of the ballistic coefficient. The best and the worst cases are given on Figure 8 and Figure 9 below.

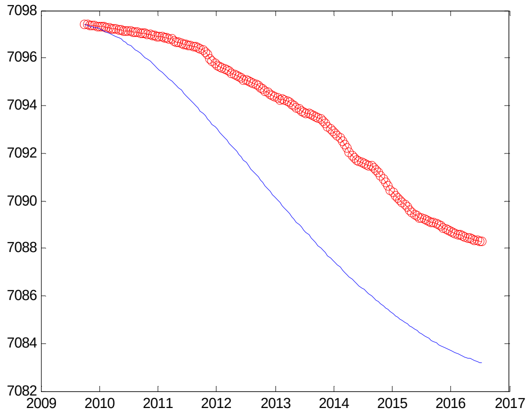

Figure 8. Worst Result using mean value (UWE-2)

For the worst case, the analysis duration is about 7 years and total semi-major axis error is smaller than 6 km at the end of the analysis period. This case is obtained from UWE-2 satellite data. The best case belongs to iCUBE-1 satellite data. Analysis duration is about 2.5 years and semi-major axis error is smaller than 1 km at the end of the analysis period.

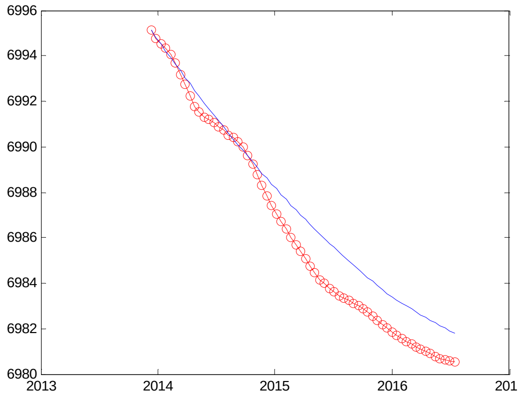

Figure 9. Best result using mean value (iCUBE-6)

## 5. Conclusion

A quick assumption can be made for CubeSat ballistic coefficient $B$ taking $C_d = 2.5$, $A = 0.1 \times 0.1 = 0.01\,m^2$, $m = 1.33\,kg$. This leads to a value $B_{standart} = 53.2$ which is different than the found mean value in this paper. This paper investigated the ballistic coefficient using real satellite data for 1U satellites. Finally, it can be proposed that usage of modified mean ballistic coefficient value $B_{mean} = 102$ with the proposed modified solar activity approximation which uses Harris-Priester model results in a better orbital decay approximation than using the $B_{standart}$ value. This shows that using the $B_{mean}$ value instead of the standard approximation (choosing a $C_d$ value between 2.2 and 2.5 and calculating the ballistic coefficient) could give better results.

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## References

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